International Journal of Advanced Trends in Engineering and Management

(ISSN 2583-7052)

pp. 25-30, April

Hamacher Operations over Bipolar Fuzzy Matrices

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ABSTRACT: In multiple sectors, bipolar fuzzy sets have recently become a thriving field. Hamacher sum, Hamacher product, Hamacher scalar multiplication $(m._h R)$ and Hamacher exponentiation $(R^{\wedge_h m})$ operations on bipolar fuzzy matrices (BFMs) defined in this article. On BFMs, also describe their algebraic properties.

Keywords: Hamacher operations, Multiplication and Exponentiation.

1. Introduction

Almost every discipline has adopted fuzzy concepts since Zadeh [11] first fuzzy sets in 1965. In 1977, Thomason [10] developed the notions of FMs. An initial concept of a bipolar fuzzy set was introduced by Zhang [12]. In 2019, M. Pal and Mondal S. [5] examined BFSs and specific findings on BFMs are reviewed. A couple of operations on Intuitionistic Fuzzy Matrices (IFMs), like the Hamacher operations are suggested and explained by Silambarasan and Sriram [8] as algebraic characteristics of IFMs. Guiwuwei et. al [3] proposed bipolar fuzzy hamacher aggregation operators (MADM). Jeong Geo Lee and Kul Hur [4] initiated the notions of BF reflexive, symmetric and transitive relation. They are notions of a BF equivalence class and BF partitions are established. Chiranjibe Jana and M.Pal [2] presented hamacher aggregation operators such as the picture fuzzy hamacher weighted averaging operator for the assessment of qualified enterprise selection. Applications of the bipolar complex fuzzy Hamacher aggregation operators in MADM were covered by Tahir Mahmood et al. [9].

2. Preliminaries

A few pertinent fundamental definitions and findings are retained for further use.

2.1 Definition [11]

The fuzzy set is defined by the membership function$\mu_F(u): u \rightarrow [0,1]$. Any value between 0 and 1 can be an element of the discourse universe U which belongs to the fuzzy set. For an element u, its degree of membership is $0 \leq \mu_F(u) \leq 1$.

2.2 Definition [10]

Take R be matrix, $R = \left[(r_{ij})\right]_{u\times v}$, where r_{ij} lies 0 and $1, 1 \leq i \leq u$ and $1 \leq j \leq v$ then R is said to FM.

2.3 Definition [4, 12]

A bipolar fuzzy set is a pair of $(-r_n, r_p)$ where $-r_n: r \rightarrow [-1,0]$ and $r_p: r \rightarrow [0,1]$ are the respectively -ve and +ve membership degree of $r \in R$. The set of all bipolar fuzzy set on R is denoted by $B_F(R)$. Bipolar fuzzy set is an extension of fuzzy set.

2.4 Definition [5]

A bipolar fuzzy matrix$R = [(r_{ij})]$ where (r_{ij}) is defined as $\langle -r_{ijn}, r_{ijp}\rangle$ whose $r_{ijn}, r_{ijp} \in$

$[0,1]\ \forall\, i,j$ represent the element's negative and positive membership values, respectively, r_{ij}.

2.5 Definition (Some special bipolar fuzzy matrices) [5]

(i) A zero bipolar fuzzy matrix is the arrangement of zero elements into rows and columns. It is a matrix with all its entries equal to zero. A matrix whose entries are all zero O_n of order $n \times n$ is the matrix of elements $O_b = (0,0)$.

(ii) A Identity bipolar fuzzy matrix I_n is an $n \times n$ matrix with the main diagonal elements are $i_b = \langle -1, 1\rangle$ and all order elements are $O_b = (0,0)$.

(iii) Let J_n be a unit bipolar fuzzy matrix whose all the entries of J_n of order $n \times n$ is the matrix of elements $\langle -1, 1\rangle$.

2.6 Definition (Functions of BFMs [5])

Let $X = [x_{ij} = \langle -x_{ijn}, x_{ijp}\rangle]_{p\times q}$, $Y = [y_{ij} = \langle -y_{ijn}, y_{ijp}\rangle]_{p\times q}$ be two bipolar fuzzy matrices.

(i) $X \vee Y = X + Y = (x_{ij} + y_{ij})_{p\times q} = (-\max\{x_{ijn,}y_{ijn}\}, max\{x_{ijp,}y_{ijp}\})_{p\times q}\ \forall\, i,j$

(ii) $X \wedge Y = X \cdot Y = (x_{ij} \cdot y_{ij})_{p\times q} = (-\min\{x_{ijn,}y_{ijn}\}, min\{x_{ijp,}y_{ijp}\})_{p\times q}\ \forall\, i,j$

Let $G = \langle -g_{ijn}, g_{ijp}\rangle \in BFM_{u\times v}$. Then G^c expand as $G^c = \langle -1 + g_{ijn}, 1 - g_{ijp}\rangle \forall i\,, j$.

3. Hamacher sum and hamacher product on BFMs.

3.1 Definition Let $\boldsymbol{R}, \boldsymbol{T} \in \boldsymbol{BF_{mn}}$,

(i) $R \oplus_{HS} T = \left(\left\langle -\left(\frac{r_{ijn}+t_{ijn}-2r_{ijn}t_{ijn}}{1-r_{ijn}t_{ijn}}\right), \frac{r_{ijp}+t_{ijp}-2r_{ijp}t_{ijp}}{1-r_{ijp}t_{ijp}}\right\rangle\right)$

(ii) $R \odot_{HP} T = \left(\left\langle -\left(\frac{r_{ijn}t_{ijn}}{r_{ijn}+t_{ijn}-r_{ijn}t_{ijn}}\right), \frac{r_{ijp}t_{ijp}}{r_{ijp}+t_{ijp}-r_{ijp}t_{ijp}}\right\rangle\right) \forall\, i,j$

3.2 Property For $\boldsymbol{R} \in \boldsymbol{BF_{mn}}$, $\boldsymbol{R} \odot_{\boldsymbol{HP}} \boldsymbol{T} \le \boldsymbol{R} \oplus_{\boldsymbol{HS}} \boldsymbol{T}$.

Proof:

$$\frac{r_{ijn}t_{ijn}}{r_{ijn}+t_{ijn}-r_{ijn}t_{ijn}} \le \frac{r_{ijn}+t_{ijn}-2r_{ijn}t_{ijn}}{1-r_{ijn}t_{ijn}}$$

$$\frac{r_{ijp}t_{ijp}}{r_{ijp}+t_{ijp}-r_{ijp}t_{ijp}} \le \frac{r_{ijp}+t_{ijp}-2r_{ijp}t_{ijp}}{1-r_{ijp}t_{ijp}}$$

Hence, ij^{th} entry of $R \odot_{HP} T \le ij^{th}$ entry of $R \oplus_{HS} T$.

Therefore, $R \odot_{HP} T \le R \oplus_{HS} T$.

3.3 Property Take $\boldsymbol{R} \in \boldsymbol{BF_{mn}}$,

(i) $R \oplus_{HS} R \ge R$,

(ii) $R \odot_{HP} R \le R$.

Proof:

(i) $R \oplus_{HS} R = \left(\left\langle -\left(\frac{2r_{ijn}-2r_{ijn}^2}{1-r_{ijn}^2}\right), \frac{2r_{ijp}-2r_{ijp}^2}{1-r_{ijp}^2}\right\rangle\right)$

$=\left(\left\langle -\left(\frac{2r_{ijn}}{1+r_{ijn}}\right), \frac{2r_{ijp}}{1+r_{ijp}}\right\rangle\right)$

$\ge (\langle -r_{ijn}, r_{ijp}\rangle) \ge R$

Since $\frac{2r_{ijn}}{1+r_{ijn}} \ge r_{ijn}$ and $\frac{2r_{ijp}}{1+r_{ijp}} \ge r_{ijp}$.

Hence, ij^{th} entry of $R \oplus_{HS} R \ge ij^{th}$ entry of R.

Therefore, $R \oplus_{HS} R \ge R$.

(ii) It is similarly provable.

These traits are easily recognized. For $\oplus_{HS}$ and $\odot_{HP}$, both commutative and associative characteristics are applicable. The identity elements' existence in connection to $\oplus_{HS}$ and $\odot_{HP}$ is demonstrated by the following theorems.

3.4 Property Take $\boldsymbol{R}, \boldsymbol{T}, \boldsymbol{S} \in \boldsymbol{BF_{mn}}$,

(i) $R \oplus_{HS} T = T \oplus_{HS} R$,

(ii) $(R \oplus_{HS} T) \oplus_{HS} S = R \oplus_{HS} (T \oplus_{HS} S)$,

(iii) $R \odot_{HP} T = T \odot_{HP} R$,

(iv) $(R \odot_{HP} T) \odot_{HP} S = R \odot_{HP} (T \odot_{HP} S)$.

3.5 Property Take $\boldsymbol{R}, \boldsymbol{T} \in \boldsymbol{BF_{mn}}$,

(i) $R \oplus_{HS} O = O \oplus_{HS} R = R$,

(ii) $R \odot_{HP} J = J \odot_{HP} R = R$,

(iii) $R \odot_{HP} O = O$,

(iv)$R \oplus_{HS} J = J$.

These two monoids are commutative: $(BF_{mn}, \oplus_{HS})$ and $(BF_{mn}, \odot_{HP})$. For transposition, operations $\oplus_{HS}$ and $\odot_{HP}$ do not adhere to De Morgan's rules.

3.6 Property Take $\boldsymbol{R}, \boldsymbol{S} \in \boldsymbol{BF_{mn}}$,

(i) $(R \oplus_{HS} S)^T = R^T \oplus_{HS} S^T$,

(ii) $(R \odot_{HP} S)^T = R^T \odot_{HP} S^T$.

Where R is transposed to R^T

3.7 Property Take $\boldsymbol{R}, \boldsymbol{T} \in \boldsymbol{BF_{mn}}$, if $\boldsymbol{R} \leq \boldsymbol{T}$, afterward $\boldsymbol{R \oplus_{HS} S \leq T \oplus_{HS} S}$.

Proof: Let $r_{ijn} \leq t_{ijn}$ and $r_{ijp} \leq t_{ijp}$ for all i, j.

$$\frac{r_{ijn} + s_{ijn} - 2r_{ijn}s_{ijn}}{1 - r_{ijn}s_{ijn}} \leq \frac{t_{ijn} + s_{ijn} - 2t_{ijn}s_{ijn}}{1 - t_{ijn}s_{ijn}}$$

$$\frac{r_{ijp} + s_{ijp} - 2r_{ijp}s_{ijp}}{1 - r_{ijp}s_{ijp}} \leq \frac{t_{ijp} + s_{ijp} - 2t_{ijp}s_{ijp}}{1 - t_{ijp}s_{ijp}}$$

Therefore, ij^{th} entry of $R \oplus_{HS} S \leq ij^{th}$ entry of $T \oplus_{HS} S$.

3.8 Property Take $\boldsymbol{R}, \boldsymbol{T} \in \boldsymbol{BF_{mn}}$, **if** $\boldsymbol{R} \leq \boldsymbol{T}$, $\boldsymbol{R \odot_{HP} S \leq T \odot_{HP} S}$.

Proof: Let $r_{ijn} \leq t_{ijn}$ and $r_{ijp} \leq t_{ijp}$ for all i, j.

$$\frac{r_{ijn}s_{ijn}}{r_{ijn} + s_{ijn} - r_{ijn}s_{ijn}} \leq \frac{t_{ijn}s_{ijn}}{t_{ijn} + s_{ijn} - t_{ijn}s_{ijn}}$$

$$\frac{r_{ijp}s_{ijp}}{r_{ijp} + s_{ijp} - r_{ijp}s_{ijp}} \leq \frac{t_{ijp}s_{ijp}}{t_{ijp} + s_{ijp} - t_{ijp}s_{ijp}}$$

Therefore, $R \odot_{HP} S \leq T \odot_{HP} S$.

3.9 Property For $\boldsymbol{R}, \boldsymbol{T} \in \boldsymbol{BF_{mn}}$,

(i) $(R \wedge T) \oplus_{HS} (R \vee T) = R \oplus_{HS} T$,

(ii) $(R \wedge T) \odot_{HP} (R \vee T) = R \odot_{HP} T$.

Proof: (i) $(R \wedge T) \oplus_{HS} (R \vee T) = (\langle min(r_{ijn}, t_{ijn}), min(r_{ijp}, t_{ijp}) \rangle) \oplus_{HS}$

$$(\langle max(r_{ijn}, t_{ijn}), max(r_{ijp}, t_{ijp}) \rangle)$$

$$= \left(\left\langle -\left(\frac{min(r_{ijn},t_{ijn}) + max(r_{ijn},t_{ijn}) - 2min(r_{ijn},t_{ijn})max(r_{ijn},t_{ijn})}{1 - min(r_{ijn},t_{ijn})max(r_{ijn},t_{ijn})}\right)\right\rangle\right),$$

$$\left(\left\langle\left(\frac{min(r_{ijp},t_{ijp}) + max(r_{ijp},t_{ijp}) - 2min(r_{ijp},t_{ijp})max(r_{ijp},t_{ijp})}{1 - min(r_{ijp},t_{ijp})max(r_{ijp},t_{ijp})}\right)\right\rangle\right)$$

$$= \left(\left\langle -\left(\frac{r_{ijn}+t_{ijn}-2r_{ijn}t_{ijn}}{1-r_{ijn}t_{ijn}}\right), \frac{r_{ijp}+t_{ijp}-2r_{ijp}t_{ijp}}{1-r_{ijp}t_{ijp}}\right\rangle\right) = R \oplus_{HS} T.$$

(ii) It is similarly provable

3.10 Property: Take $\boldsymbol{R}, \boldsymbol{T} \in \boldsymbol{BF_{mn}}$,

(i) $(R \oplus_{HS} T)^c = R^c \odot_{HP} T^c$,

(ii) $(R \odot_{HP} T)^c = R^c \oplus_{HS} T^c$.

4. Hamacher scalar multiplication $(m._h R)$ and Hamacher exponentiation $(R^{\wedge_h m})$.

This part, $(m._h R)$ and $(R^{\wedge_h m})$ on BFMs illustrated. Some qualities over BFMs are obtained.

$$m._h R:$$

Utilizing the meaning of Hamacher sum $R \oplus_{HS} T$,

$$R._h R = \left(\left\langle -\left(\frac{r_{ijn} + r_{ijn} - 2r_{ijn}r_{ijn}}{1 - r_{ijn}r_{ijn}}\right), \frac{r_{ijp} + r_{ijp} - 2r_{ijp}r_{ijp}}{1 - r_{ijp}r_{ijp}}\right\rangle\right)$$

$$= \left(\left\langle -\left(\frac{2r_{ijn} - 2r_{ijn}^2}{1 - r_{ijn}^2}\right), \frac{2r_{ijp} - 2r_{ijp}^2}{1 - r_{ijp}^2}\right\rangle\right)$$

$$\left(\left\langle -\left(\frac{2r_{ijn}}{1 + r_{ijn}}\right), \frac{2r_{ijp}}{1 + r_{ijp}}\right\rangle\right)$$

$$2._h R = \left(\left\langle -\left(\frac{2r_{ijn}}{1 + (2-1)r_{ijn}}\right), \frac{2r_{ijp}}{1 + (2-1)r_{ijp}}\right\rangle\right)$$

$$3._h R = \left(\left\langle -\left(\frac{3r_{ijn}}{1 + (3-1)r_{ijn}}\right), \frac{3r_{ijp}}{1 + (3-1)r_{ijp}}\right\rangle\right)$$

In overall,

$$m._h R = \left(\left\langle -\left(\frac{mr_{ijn}}{1 + (m-1)r_{ijn}}\right), \frac{mr_{ijp}}{1 + (m-1)r_{ijp}}\right\rangle\right)$$

Hamacher exponentiation $(R^{\wedge_h m})$.

Utilizing the meaning of Hamacher product $R \odot_{Hp} T$,

$$R^{\wedge_h R} = \left(\left\langle -\left(\frac{r_{ijn} r_{ijn}}{r_{ijn} + r_{ijn} - r_{ijn} r_{ijn}}\right), \frac{r_{ijp} r_{ijp}}{r_{ijp} + r_{ijp} - r_{ijp} r_{ijp}}\right\rangle\right)$$

$$= \left(\left\langle -\left(\frac{r_{ijn}^2}{2r_{ijn} - r_{ijn}^2}\right), \frac{r_{ijp}^2}{2r_{ijp} - r_{ijp}^2}\right\rangle\right)$$

$$= \left(\left\langle -\left(\frac{r_{ijn}}{2 - r_{ijn}}\right), \frac{r_{ijp}}{2 - r_{ijp}}\right\rangle\right)$$

$$R^{\wedge_h 2} = \left(\left\langle -\left(\frac{r_{ijn}}{2 - (2-1) r_{ijn}}\right), \frac{r_{ijp}}{2 - (2-1) r_{ijp}}\right\rangle\right)$$

$$R^{\wedge_h 3} = \left(\left\langle -\left(\frac{r_{ijn}}{3 - (3-1) r_{ijn}}\right), \frac{r_{ijp}}{3 - (3-1) r_{ijp}}\right\rangle\right)$$

In overall,

$$R^{\wedge_h m} = \left(\left\langle -\left(\frac{r_{ijn}}{m - (m-1) r_{ijn}}\right), \frac{r_{ijp}}{m - (m-1) r_{ijp}}\right\rangle\right)$$

4. 1 Property: Take $\boldsymbol{R}, \boldsymbol{T} \in \boldsymbol{BF_{mn}}$, and +ve integers $\boldsymbol{m}, \boldsymbol{m_1}, \boldsymbol{m_2}$.

(i) $m_{1.h} R \oplus_{HS} m_{2.h} R = (m_1 + m_2)_{.h} R$,

(ii) $(m._h R) \oplus_{HS} (m._h T) = m_{.h}(R \oplus_{HS} T)$,

(iii) $R^{\wedge_h m_1} \odot_{HP} R^{\wedge_h m_2} = R^{\wedge_h (m_1 + m_2)}$,

(iv) $R^{\wedge_h m} \odot_{HP} T^{\wedge_h m} = (R \odot_{HP} T)^{\wedge_h m}$,

(v) $m_{2.h}(m_{1.h} R) = (m_1 m_2)_{.h} R$,

(vi) $(R^{\wedge_h m_1})^{\wedge_h m_2} = R^{\wedge_h (m_1 m_2)}$.

4. 2 Property: Take $\boldsymbol{R}, \boldsymbol{T} \in \boldsymbol{BF_{mn}}$, and +ve integer $\boldsymbol{m}$.

(i) $m_{.h}(R \wedge T) = (m_{.h} R) \wedge (m_{.h} T)$,

(ii) $m_{.h}(R \vee T) = (m_{.h} R) \vee (m_{.h} T)$,

(iii) $(R \wedge T)^{\wedge_h m} = R^{\wedge_h m} \wedge T^{\wedge_h m}$,

$(R \vee T)^{\wedge_h m} = R^{\wedge_h m} \vee T^{\wedge_h m}$

5. Application of Bipolar fuzzy matrices in decison making

In this section, we present one example of a decision making based on score value. Finally, selecting the best purifier for the good health of humans.

Methodology:

Step: 1 To determine the BFSs over D

Step: 2 To determine the BFMs an R and T

Step: 3 To evaluate

Step: 4 To examine the total score S_i and $S_k = \max(S_i)$ and afterward choose the best option purifier S_k has the maximum value.

Example: Suppose there are three purifiers $P = \{p_1, p_2, p_3\}$ and set D is made up of parameters related to the purifier. Finally selecting the best purifier for the good health of humans.

Step: 1

Consider the following three Bipolar fuzzy sets are given by, $(F, R) = F_R(d_1), F_R(d_2), F_R(d_3)$

Where, $F_R(d_1) = \{(p_1, -0.6, 0.5), (p_2, -0.2, 0.3), (p_3, -0.3, 0.4)\}$

$F_R(d_2) = \{(p_1, -0.4, 0.1), (p_2, -0.3, 0.5), (p_3, -0.7, 0.2)\}$

$F_R(d_3) = \{(p_1, -0.2, 0.3), (p_2, -0.2, 0.4), (p_3, -0.6, 0.2)\}$

$(F, T) = \{F_T(d_1), F_T(d_2), F_T(d_3)\}$ Where,

$F_T(d_1) = \{(p_1, -0.4, 0.2), (p_2, -0.5, 0.1), (p_3, -0.4, 0.3)\}$

$F_T(d_2) = \{(p_1, -0.3, 0.4), (p_2, -0.6, 0.3), (p_3, -0.5, 0.2)\}$

$F_T(d_3) = \{(p_1, -0.1, 0.4), (p_2, -0.2, 0.6), (p_3, -0.1, 0.3)\}$

Step: 2

Construction of Bipolar fuzzy matrices

$$R = \begin{bmatrix} (-0.6, 0.5) & (-0.4, 0.1) & (-0.2, 0.3) \\ (-0.2, 0.3) & (-0.3, 0.5) & (-0.2, 0.4) \\ (-0.3, 0.4) & (-0.7, 0.2) & (-0.6, 0.2) \end{bmatrix}$$

$$T = \begin{bmatrix} (-0.4, 0.2) & (-0.3, 0.4) & (-0.1, 0.4) \\ (-0.5, 0.1) & (-0.6, 0.3) & (-0.2, 0.6) \\ (-0.4, 0.3) & (-0.5, 0.2) & (-0.1, 0.3) \end{bmatrix}$$

$$R^C = \begin{bmatrix} (-0.4, 0.5) & (-0.6, 0.9) & (-0.8, 0.7) \\ (-0.8, 0.7) & (-0.7, 0.5) & (-0.8, 0.6) \\ (-0.7, 0.6) & (-0.3, 0.8) & (-0.4, 0.8) \end{bmatrix}$$

$$T^C = \begin{bmatrix} (-0.6, 0.8) & (-0.7, 0.6) & (-0.9, 0.6) \\ (-0.5, 0.9) & (-0.4, 0.7) & (-0.8, 0.4) \\ (-0.6, 0.7) & (-0.5, 0.8) & (-0.9, 0.7) \end{bmatrix}$$

Step: 3 To calculate $R \odot_{HP} T, R^c \odot_{HP} T^c, V(R \odot_{HP} T), V(R^c \odot_{HP} T^c)$

And $S_{((R \odot_{HP} T)(R^c \odot_{HP} T^c))}$

$$R \odot_{HP} T = \begin{bmatrix} (-0.32, 0.17) & (-0.21, 0.09) & (-0.07, 0.21) \\ (-0.17, 0.08) & (-0.25, 0.23) & (-0.11, 0.32) \\ (-0.21, 0.21) & (-0.41, 0.11) & (-0.09, 0.14) \end{bmatrix}$$

$$R^c \odot_{HP} T^c = \begin{bmatrix} (-0.32, 0.44) & (-0.48, 0.56) & (-0.73, 0.48) \\ (-0.44, 0.65) & (-0.34, 0.41) & (-0.67, 0.32) \\ (-0.48, 0.48) & (-0.23, 0.67) & (-0.38, 0.60) \end{bmatrix}$$

$$V(R \odot_{HP} T) = \begin{bmatrix} -0.49 & -0.3 & -0.28 \\ -0.25 & -0.48 & -0.43 \\ -0.42 & -0.52 & -0.23 \end{bmatrix}$$

$$V(R^c \odot_{HP} T^c) = \begin{bmatrix} -0.76 & -1.04 & -1.21 \\ -1.09 & -0.75 & -0.99 \\ -0.96 & -0.9 & -0.98 \end{bmatrix}$$

$$S_{((R \odot_{HP} T)(R^c \odot_{HP} T^c))} = \begin{bmatrix} 0.27 & 0.74 & 0.93 \\ 0.84 & 0.27 & 0.56 \\ 0.54 & 0.38 & 0.75 \end{bmatrix}$$

Step:4

$$\text{Total score} = \begin{bmatrix} 1.94 \\ 1.67 \\ 1.67 \end{bmatrix}$$

$p_1 = 1.94$ is maximum value.

Therefore, p_1 is the best purifier.

6. Conclusion

In this article, we found Hamacher sum, Hamacher product, Hamacher scalar multiplication $(m._h R)$ and Hamacher exponentiation $(R^{\wedge_h m})$ operations on BFMs. Additionally, certain algebraic properties are demonstrated on BFMs.

References

1. Anam Luqman; Gulfam Shahzadi, Year: 2023, "Multi-attribute decision making for electronic waste recycling using interval-valued fermatean fuzzy hamacher aggregation operators", Granular computing-springer, Vol: 8, No: 5, pp. 991-1012.

2. Chiranjibe Jana; Madhumangal Pal, Year: 2019, "Assessment off enterprise performance based on picture fuzzy hamacher aggregation operators", Symmetry, Vol: 11, No: 1, pp. 75.

3. Guiwu Wei; Fuad E. Alsaadi; Tasawar Hayat; Ahmed Alsaedi, Year: 2018, "Bipolar fuzzy hamacher aggregation operators in multiple attribute decision making", International Journal of Fuzzy Systems, No: 20, pp. 1-12.

4. Jeong-Gon Lee; Kul Hur, Year: 2019, "Bipolar fuzzy relation", Fuzzy sets, Fuzzy Logic and their Applications, Vol: 7, No: 11, pp.1044.

5. Madhumangal Pal; Sanjib Mondal, Year: 2019, "Bipolar fuzzy matrices", Soft Computing, Vol: 23, No: 20, pp. 9885-9897.

6. I. Silambarasan; S. Sriram, Year: 2019, "Hamacher operations on Pythagorean fuzzy matrices", Journal of Applied Mathematics and Computational Mechanics, Vol: 18, No: 3.

7. I. Silambarasan; S.Sriram, Year: 2020, "Some operations over Pythagorean Fuzzy Matrices based on Hamacher operations", An International Journal of Applications and Applied Mathematics, Vol. 15, No: 1, pp.20.

8. I. Silambarasan; S. Sriram, Year: 2018, "Hamacher operations of Intuitionistic Fuzzy Matrices", Annals of Pure and Applied Mathematics, Vol: 16, No: 1, pp.81-90.

9. Tahir Mahood; Ubaid ur Rehman; Jabbar Ahmmadand; Gustavo Santos-Garcia, Year: 2021, “Bipolar complex hamacher aggregation operators and their applications in multi-attribute decision making”, Mathematics, Vol: 10, No: 1, pp.23.

10. Michael G Thomason, Year: 1977, “Convergence of powers of a fuzzy matrix”, J. Math. Anal. Appl., Vol: 57, No: 2, pp. 476-480.

11. Lotfi A. Zadeh, Year: 1965, “Fuzzy sets”, Journal of Information and Control, Vol: 8, No: 3, pp. 338-353.

12. Wen-Ran Zhang, Year: 1994, “Bipolar fuzzy sets and relations”, First International Joint Conference of The North American Fuzzy Information Processing Society Biannual Conference, pp. 305-309.